ENGINEERING DEPARTMENT
General Radio Company
CAMBRIDGE, MASSACHUSETTS

Reprint No. A-78 — 1959

Reprinted from IRE TRANSACTIONS ON INSTRUMENTATION — December, 1958

New Apparatus at the National Bureau of Standards for Absolute Capacitance Measurements

BY

M. C. McGREGOR, J. F. HERSH, R. D. CUTKOSKY, F. K. HARRIS AND F. R. KOTTER

PRINTED IN U.S.A.

New Apparatus at the National Bureau of Standards for Absolute Capacitance Measurement*

M. C. McGREGOR†, J. F. HERSH‡, R. D. CUTKOSKY§, F. K. HARRIS§, AND F. R. KOTTER§

Introduction

THE use of tightly coupled inductive ratio arms rather than resistive ratio arms in a 4-arm bridge for the comparison of impedances was suggested by Blumlein[1] in 1928, and the use of a 3-winding transformer in such a bridge circuit was described by Starr[2] in 1932. Other bridges using Blumlein's principle have been described by several workers[3] in the past 30 years. Historically, it is of interest to note that conjugate bridges making use of 3-winding transformers were described by Elsas[4] in 1888 for resistance comparison and by Trowbridge[5] in 1905 for capacitance and inductance comparisons.

Thus the basic principle of operation and the general arrangement of transformer bridges have been known for many years. However, the possibilities of such bridges for the precise comparison of very low value capacitors had never been fully exploited before the work of Thompson and his group at the National Standards Laboratory of Australia. By combining the best techniques for constructing ratio transformers, completely shielded 3-terminal capacitors and detectors of high sensitivity, together with a cylindrical cross capacitor as a calculable standard, there is now promise of being able to assign values to capacitance standards comparable with, or perhaps even better than, the accuracy assigned to our present standards of electromotive force and resistance.

The present paper describes a transformer bridge constructed at the National Bureau of Standards for measuring the direct capacitance of 3-terminal capacitors ranging in values up to 1 μf and having a least count of 1 μpf. Although the transformers and network components described below were designed specifically for operation at 1 kc, the operation is by no means limited to this frequency. Voltage output of the ratio transformers constitutes the most serious limitation at lower frequencies, but it is reasonable to suppose that, with relatively minor modifications, satisfactory operation should be possible over the audio-frequency range to at least 10 kc.

While some of the present bridge components differ substantially from their counterparts at NSL, it should be understood that no more is involved generally than modifications and in some cases improvements of designs already proven by Thompson and his group in Sydney.

There has been little detailed information published up to now concerning these components, and the present paper must be considered primarily as a discussion of the constructional details and performance of the NBS transformer-ratio bridge.

* Manuscript received by the PGI, August 14, 1958.
† National Standards Lab., Chippendale, N.S.W., Australia.
‡ General Radio Co., Concord, Mass.
§ National Bureau of Standards, Washington, D. C.

[1] British Patent No. 323037.

[2] A. T. Starr, "A note on impedance measurement," *W. Eng. and Exp. W.*, vol. 9, pp. 615–617; November, 1932.

[3] C. H. Young, "Measuring inter-electrode capacitances," *Bell Labs. Rec.*, vol. 24, pp. 433–438; December, 1946.
H. A. M. Clark and P. B. Vanderlyn, "A. C. bridges with inductively coupled ratio arms," *Proc. IEE*, vol. 96, pp. 365–378; May, 1949.
C. W. Oatley and J. G. Yates, "Bridges with coupled inductive ratio arms for the comparison of standards of resistance or capacitance," *Proc. IEE*, vol. 101, pp. 91–100; March, 1954.
A. M. Thompson, "A bridge for the measurement of permittivity," *Proc. IEE*, vol. 103, pt. B, pp. 704–707; November, 1956.

[4] A. Elsas, "Ueber Widerstandsmessungen mit dem Differentialinductor," *Ann. Phys.*, vol. 35, pp. 828–833; 1888.

[5] A. Trowbridge, "On the differential transformer," *Phys. Rev.* vol. 20, pp. 65–76; 1905.

Balance Conditions

The transformer bridge with closely coupled ratio arms may be considered as an arrangement of two low-impedance generators supplying EMF's of opposite phase, and of a known ratio, to two completely shielded 3-terminal capacitors as shown schematically in Fig. 1(a). The capacitances C_{l_1}, C_{l_2} between the enclosing shields and the line terminals l_1, l_2 of capacitors C_1, C_2 will be in shunt across one or the other of the two generators, as shown in Fig. 1(b). The capacitances C_{d_1}, C_{d_2} between shields and detector terminals d_1, d_2 of C_1, C_2 will be in shunt across the detector. Hence, when the bridge is balanced and there is no current through the detector, the currents through the direct capacitances C_1, C_2 must be equal in magnitude, and the balance relation must be

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}.$$

If now the impedances Z_1, Z_2 are negligible compared to

$$\frac{1}{\omega C_{l_1}}, \quad \frac{1}{\omega C_{l_2}},$$

then, to a very close approximation,

$$\frac{e_1}{e_2} = \frac{C_2}{C_1}.$$

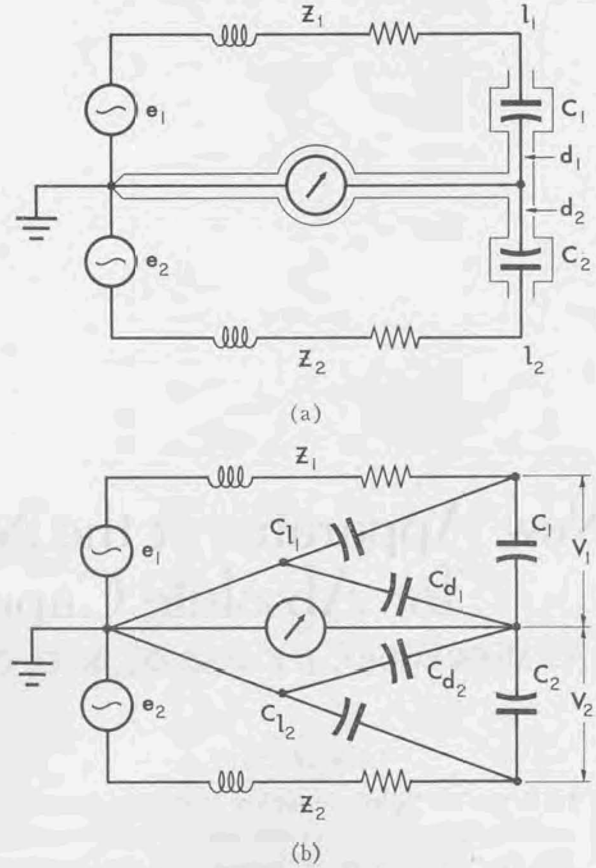

Fig. 1—Transformer-ratio bridge schematic.

Transformer Design and Construction

By careful design of the transformer, the impedances Z_1, Z_2 of its ratio arms can be made very small, and the ratio of the induced voltages e_1/e_2 can be made stable and precise. In a 3-winding transformer, arranged as in Fig. 2, the sole function of the primary winding P is to provide flux in the common core, linking the secondary windings S_1, S_2. The ratio of the terminal voltages V_1/V_2 of the ratio arms under load is affected by the resistances and leakage reactances of the secondary windings, but the resistance and leakage reactance associated with the primary will in no way affect the bridge balance relation. If the two secondary windings could be so constructed that a common flux were confined entirely within the windings, the ratio of their induced voltages would depend only upon their turns ratio, and this ratio would be stable and exact. If, in addition, the elements of the two windings S_1, S_2 could be brought into exact coincidence, turn by turn, so as to link identical flux at any loading, the series impedances, producing changes in the terminal voltages V_1, V_2 when loads are applied, would be only the resistances of the windings.

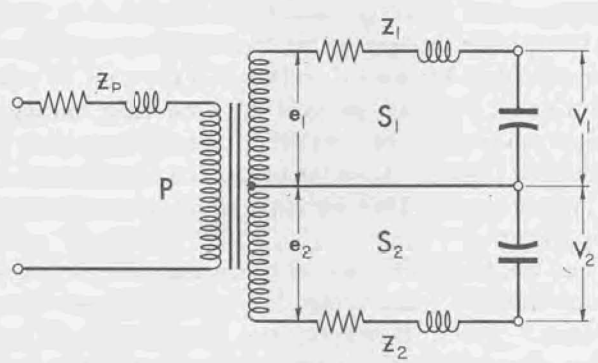

Fig. 2—Equivalent circuit of a loaded 3-winding transformer.

The loading error of the bridge ratio can be made small by decreasing the size of the resistances of the secondary windings and by reducing as far as possible their effective leakage inductances. It should be observed that the self-capacitance of the secondary windings of the transformer constitutes a load which is always present, so that the terminal voltages of the transformer differ from the induced voltages even on open circuit. The amount of this difference and, even more important, the amount by which the open-circuit ratio differs from the turns ratio will depend on the magnitudes and distribution of these self-capacitances. The transformer design should be such as to keep the self-capacitances of the secondaries as small as possible, and ideally these capacitances should be symmetrically distributed.

In the design and construction of the transformers used for ratio arms of the bridge, the features favorable to good performance have been carefully considered. Very small ratio errors, even for large loads, have been achieved by special construction. A good approximation to unity coupling has been obtained by the use of a large toroidal core of high permeability. Voltages from sources other than the core flux have been excluded by carefully shielding the secondary from the primary and from external fields, and the effect of leakage flux on the ratio has been minimized by a uniform distribution of each winding completely around the toroid. The loading error resulting from the resistance of the windings has been kept low by using copper strip rather than wire for the secondaries.

Two bridge transformers of similar design have been built, differing in some details and in the ratios of their secondary voltages. The first of these was a 10/1 ratio transformer and its construction will be considered in detail. Its core is a toroid, wound of continuous 1-mil Supermalloy tape,[6] and has an OD of 4.5 inches, an ID of 3 inches, and a height of 1.5 inches. This Supermalloy has an initial dc permeability of about 75×10^3, and the thinness of the tape permits operation at frequencies of the order of 1 kc without serious diminution of its permeability. The large cross-sectional area of the core, $1\frac{1}{8}$ square inches, enables the required secondary voltage to be obtained with few turns and, hence, with low winding resistance. When operated at 1 kc in its region of maximum permeability (flux density about 5 kilogausses) it produces about 1.6 volts per turn. A reasonable bridge voltage is of the order of 100 volts, which is obtained with a secondary winding of 70 turns. This gives good sensitivity without excessive voltage on the capacitors.

The primary winding is of rectangular copper wire 0.036 inch $\times$ 0.104 inch so that 73 turns just fill the inner circumference of the toroid. This heavy wire was selected, not from any concern for low primary resistance, but to insure a uniform and nearly continuous distribution of magnetomotive force along the toroid. Small flexible leads are connected with minimum separation to the ends of the primary winding, and one of the leads is returned around the circumference of the toroid in such a direction as to cancel the field produced by the "single-turn" effect of the toroidal winding.

It has already been pointed out that the ratio of voltages induced in the secondary windings, which is of interest in a bridge transformer, must be as completely independent of the primary as possible. To insure this independence, the secondary is electrostatically shielded from the primary by two copper shields. The inner shield is a toroidal cup of 20-mil copper fitting snugly over the primary and closed by a copper lid soldered to the cup around the outer circumference. An insulated gap along the inner edge prevents a shorted turn. One of the primary leads is connected to this shield, and the other is brought out through a small copper tube soldered to the lid. This tube and the wire inside constitute a coaxial pair of leads to the primary winding. After insulation with Teflon tape the inner shield is fitted into another toroidal cup which is closed with a similar lid having an insulated gap around its inner circumference. Connection to the outer shield is by a tube coaxial with the primary-lead tube but insulated from it. The outer shield is covered with an impregnated glass-fiber insulation which can withstand the heat and mechanical abrasion involved in assembling the secondary windings. The use of two separate shields between primary and secondary makes it possible to separate the grounds so that, even with considerable capacitance current from the primary to the inner shield, the outer shield, next to the secondary, can be kept at the ground potential of the bridge. In addition the double copper shield provides some magnetic shielding as a result of eddy current action and so assists in confining the primary flux, with the result that all the secondary turns more nearly link all the primary flux than would be the case without this shielding.

The two secondaries which provide bridge voltages of precise and stable ratio are wound around the shielded and electrostatically isolated primary. As noted above, a winding of about 70 turns provides the desired 100-volt output. For a 1:1 ratio the secondary winding would consist of two sections of 70 turns each. The coupling of these sections to the primary could be made nearly identical by the familiar use of a bifilar winding in which corresponding turns of the two sections are made to occupy, as nearly as possible, the same position relative to the core. Any nonuniformity in flux distribution around the core would then influence both sections alike, and the ratio of secondary voltages would not be affected. For ratios other than unity the winding cannot be truly bifilar, but the turns of the two sections can be distributed to effect a similar sampling of the flux around the core by both. The secondary windings in the 10:1 ratio transformer are distributed in the following manner. The secondary is divided into twenty 7-turn sections. The seven turns of each section are wound as a uniform spiral around the toroid so that each section samples the flux around the entire toroid and ends at a point adjacent to its beginning. The ends of all 20 sections are brought out at the outer circumference in such a manner that 10 sections can be connected in series to form a 70-turn winding, and the other 10 sections in parallel to form a 7-turn winding. Further, the connections are such that the turns of the 70-turn winding are always separated by turns of the 7-turn winding as they progress around the toroid, thus approximating the uniform flux sampling of a bifilar winding.

The requirement of low error in the secondary ratio, when the transformer is heavily loaded, necessitates a winding resistance of only a few milliohms. To achieve this, the cross section of the copper must be large, so that edge-wound strips rather than wire are used in the secondary to utilize best the limited space inside the toroid. The requirement that 140 turns of minimum resistance separated by 2-mil mica strips must be fitted into the toroid leads to dimensions of 0.040 inch $\times$ 0.255 inch for the cross section of the strips inside the toroid. It is convenient to use stock of 40-mil thickness for all

[6] The authors greatly appreciate the help of J. E. Mitch, chief engineer of the Arnold Eng. Co., Marengo, Ill., in selecting and making available the core material.

the winding strips, but the width of the other three sides of the turn can be increased to $\frac{1}{2}$ inch to reduce the resistance. The turns are constructed from this copper strip by using a *C*-shaped piece to enclose the top, inside, and bottom of the toroid and curved strips fitting the contour of the outer circumference to space the *C*'s at intervals of $\frac{1}{7}$ of the circumference (see Fig. 3). The strips are connected by soldering with a special clamp-type resistance heater having graphite jaws thin enough to fit the small gaps between turns. After assembly the strips inside the toroid are separated by insulating strips of 2-mil mica and on the outer circumference by strips of 6-mil impregnated glass-fiber insulation.

The transformer is enclosed in a Mu-metal can with the ends of all 20 sections of the secondary, and the coaxial primary and shield leads, brought outside the can through slots. The connections of the secondary sections in series and in parallel are made with additional copper strip outside the can, where the stray field of the transformer should be negligible. An outer brass case covers the Mu-metal shield and connections and is used to mount the BPO[7] connectors through which the transformer leads are brought out to the bridge and driving amplifier.

The characteristics of the transformer can be determined in the capacitance bridge by the intercomparison and summation of a group of closely matched capacitors, as described by Thompson.[8] The ratio error (for light loads) was measured using 100-pf (picofarad) capacitors. The ratio, defined as $V_{10}/V_1 = 10(1+\alpha+j\beta)$, has a magnitude correction $\alpha = 0.3\times10^{-6}$ and a phase correction $\beta = 2.7\times10^{-6}$. The effective leakage inductance and series resistance of the two secondary windings have been determined by measuring the ratio change when known loads are connected across one section. The equivalent circuit thus determined at the BPO connectors to the transformer is shown in Fig. 4.

A second transformer, having a toroidal core of a somewhat smaller cross section (0.94 in^2) and with an 80-turn primary, was built in a similar manner. This transformer has two equal 80-turn secondaries in 8-turn sections around the toroid, which are intermingled as described above. The ratio of this transformer at 1 kc has a magnitude correction at light loading of 0.1×10^{-6}, and a phase angle of 0.8 μr. The leakage inductance of each secondary winding is about 1 μh and its resistance is 0.030 ohm.

Design and Construction of Capacitors

An unknown capacitor supplied by one secondary of the transformer (see Fig. 1) may be balanced in the bridge by a capacitance of appropriate magnitude supplied by the other secondary. This is accomplished through the use of two 3-terminal decade capacitance

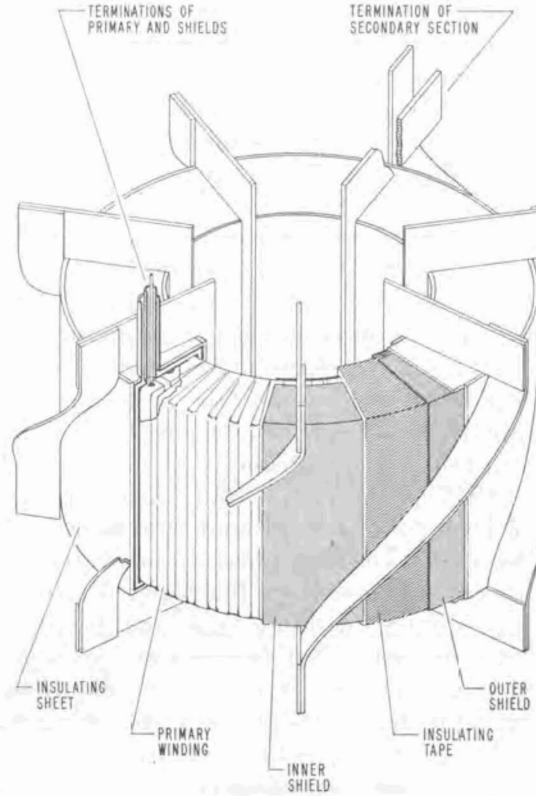

Fig. 3—Transformer construction details.

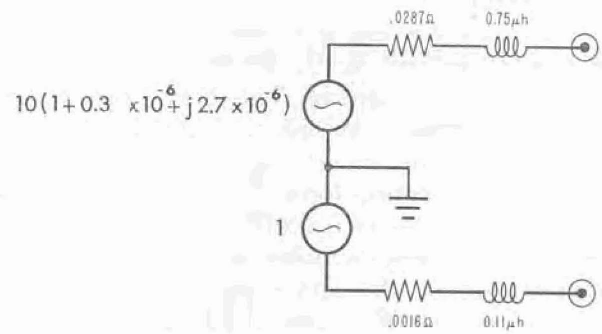

Fig. 4—10/1 transformer characteristics.

boxes covering a range of 12 orders of magnitude from 10^{-6} pf to 1 μf.

The range from 1 pf to 1 μf is covered in a 6-decade box having a special switching system that allows the selection of any individual capacitor separately for calibration and the addition of the capacitors in parallel to balance an arbitrary unknown. The switching system is such that each capacitor is either connected directly between line and detector, or is grounded at both terminals. The switch decks controlling line and detector connections, respectively, are completely shielded from each other, so that switch and lead capacitances are connected to the shield system rather than to the other active electrode of the capacitor [see Fig. 1(b)]. With this system and with the complete shielding incorporated in the design, the residual direct capacitance with all switches on zero is not observable and is certainly less than 10^{-6} pf. Each decade consists of ten equal capacitors, permitting quick intercomparison of

[7] These coaxial connectors are of British manufacture and may be described as British Post Office Pattern 8.

[8] A. M. Thompson, "The precise measurement of small capacitances," this issue, pp. 245–253.

the various units in the decade. The arrangement is also convenient in measuring the ratios of transformers used in the bridge itself, or in other ratio devices.

The upper three decades of the high-range box consist of precision-quality silvered-mica capacitors, covering the range 0.001 to 1 μf. The lower three decades, covering the range 1 pf to 10^3 pf, are air capacitors of a cylindrical design of McGregor.

The 10-pf unit is typical of the air capacitors and is shown in cross section in Fig. 5. The center post A is grounded, and the entire assembly is fastened in a hole in a grounded brass block by means of the flange. The brass block serves both as a shield and as a heat sink to minimize the rate of temperature variation in the capacitor. Cylinder B is connected by means of a wire through a hole in the ground post to the detector junction of the bridge. It is mounted on the ground post by means of two high-density polyethylene washers between the detector electrode B and the ground post. These washers are machined about 0.002 inch oversize, making an interference fit which compresses the washers and results in a rigid and mechanically stable assembly. Electrode C is also supported on polyethylene washers and is connected to a line terminal of the bridge. It should be noted that no solid insulation appears in the fieldbetween the electrodes B and C of the direct capacitance measured. In addition, the electrodes are goldplated to reduce losses arising from oxide films on their surfaces. In the absence of both solid dielectric and surface oxide films, the phase defect of these capacitors should be stable and quite small.

Fine adjustment of the capacitance between electrodes B and C is accomplished by drilling two holes on a diameter of electrode C. Field from electrode B passes through these holes and terminates on the outer shield, decreasing the direct capacitance between B and C and slightly increasing the shield capacitance to B. A sleeve with matching holes fits over electrode C, and rotation of this sleeve with respect to the electrode opens or closes the effective aperture through which electrode B "sees" the outer shield, thus changing the capacitance B–C by a small amount depending on the size of the holes.

The 1-pf unit has essentially the same construction but is shorter. The means of fine adjustment of its value is similar. The 100-pf unit makes use of a group of nested coaxial cylinders of close spacing in order to obtain the larger capacitance within approximately the same physical volume. Here the fine adjustment is accomplished by a sliding skirt, fitted to the shield, which approaches the outer active electrode and intercepts field, that would otherwise contribute to the direct capacitance, at its end. Thus the direct capacitance of each of the air capacitors is smoothly and continuously adjustable over a small range, making it possible to match closely the values of all the units in a decade. Because of the symmetrical design, small departures from radial alignment have very little effect on the capacitance of the unit, and the capacitors can be taken apart and reassembled without appreciable change in their values.

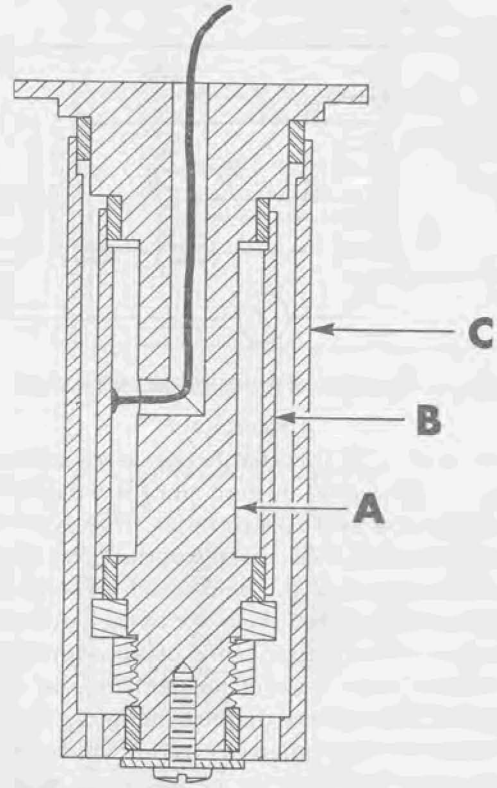

Fig. 5—Section of 10-pf capacitor.

The low-range decade box (10^{-6} pf to 1 pf) differs from the higher decades in that it uses fixed value capacitors connected to a variable voltage. This voltage is obtained from a tapped inductor, which is connected across one side of the bridge transformer to divide the transformer voltage into 10 equal steps. This tapped inductor is constructed by winding 50 turns of 10-wire tape of 20 AWG wire on a small Supermalloy core, and its impedance is high enough to impose negligible loading on the bridge transformer. If a capacitor of a given value is connected between the first tap of the divider and the detector junction point, the current which it injects into the junction will be only 1/10 of the value it would have been had the capacitor been connected directly across the transformer secondary. On the nth tap, its effect on the bridge balance will be $n/10$ of its actual value. The low-range (or micropicofarad) capacitance box consists of six separate capacitors of values, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, and 1 pf, switched independently along the tapped inductor. The entire range of 6 decades, from 10^{-6} pf to 1 pf, thus consists of a single tapped inductor, 6 switches, and 6 capacitors. The arrangement with its shielding is shown in Fig. 6.

The six capacitors have cylindrical electrodes spaced with polyethylene washers. The 1-pf unit is like the 1-pf capacitors in the high-range box. All the lower valued capacitors are of the Zichner diaphragm type, having a grounded cylindrical shield extending between the two

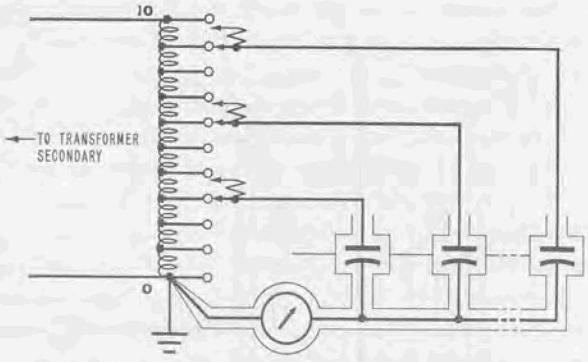

Fig. 6—Schematic of low-range decade capacitor.

active electrodes. If this diaphragm were solid, the direct capacitance would be zero since it would intercept all the field between the electrodes. However two diametrically opposite holes of appropriate size are drilled in the diaphragm, and the field which penetrates these holes and continues from the inner to the outer electrode produces the direct capacitance of appropriate magnitude. Another larger pair of holes, drilled in the outer electrode, in line with the diaphragm holes, allows some of the field from the inner electrode, which penetrates the diaphragm openings, to continue on through the openings in the outer electrode and terminate on an outer grounded shield. This decreases the value of the direct capacitance. Rotation of the outer electrode with respect to the diaphragm moves the sets of holes with respect to each other and constitutes the fine adjustment of capacitance for the unit.

The switches for the low-range decades deserve special mention because of their linear rather than rotary motion and because of the special moving-contact brushes that are used. A linear motion was preferred over the usual rotary motion for these selector switches to facilitate their manipulation either singly or in groups. This feature is particularly useful in the lower decades, as it makes possible a more rapid approach to the bridge balance. The use of these switches with an inductive voltage divider requires that special consideration be given the problem of making and breaking contact. Clearly, whatever switch is used for this application should not be of the shorting type, or a section of the inductor will be momentarily shorted during switching; nor should it be of a nonshorting type, or the detector will be momentarily floating, causing the indicating device to go off scale and perhaps causing the preamplifier to overload. The switches can be considered as nonshorting with an added moving contact brush adjacent to the main brush, the two being connected by a resistor of appropriate value (see Fig. 6). In the normal position the auxiliary brush is floating. When the switch is advanced to another position, the auxiliary brush touches the adjacent switch stud before the main contact is broken, and the main brush touches the new stud before the auxiliary contact is broken. Thus there is always a connection either direct or through the resistor, and the inductor section is at no time shunted by an impedance less than that of the resistor.

The capacitors in the units so far described are sealed to reduce changes in value resulting from variations of humidity and atmospheric pressure. Under these circumstances the principal cause of short-term drift in their values is variation in ambient temperature, as their temperature coefficients of capacitance are all nominally equal to the coefficient of linear expansion of brass, approximately $20\times10^{-6}/°C$.

Temperature-Compensated Capacitors

Standards for maintaining the unit of capacitance must be independent of environmental conditions. The first stable capacitors completed at NBS were a set of four temperature-compensated modifications of the 1-pf design used in the decade boxes. The cylindrical structure of this design was lengthened, and grounded Duralumin[9] sleeves were inserted between the active electrodes from either end of the assembly, leaving a gap in the center that determined the active length of the capacitor. The remainder of the brass capacitor structure had a linear temperature coefficient substantially less than that of the Duralumin. The length of the sleeves were so chosen that the capacitance was independent of temperature. The structure of the capacitor is shown in section in Fig. 7.

The completed capacitors have temperature coefficients in the neighborhood of 1 ppm (part per million) per °C. They are quite sensitive to temperature gradients and must be well isolated from rapid temperature changes to exhibit this low coefficient. In the final assembly these capacitors are mounted in a massive brass container with $\frac{3}{4}$-inch walls and sealed in an atmosphere of dry nitrogen. This container is surrounded with heat-insulating material and is mounted in a box. During the first month after their completion, the values of three of these capacitors did not drift relative to each other by as much as 1 ppm. The behavior of the fourth unit has been somewhat less satisfactory. It has changed by nearly 3 ppm during the same period.

Stable capacitors with low temperature coefficients covering the range from 10 to 10^4 pf are being built but have not yet been completed.

Conductance-Balance Control

At a single frequency any physically realizable capacitor may be represented by the parallel combination of pure capacitance and pure resistance. The current through it, in response to an impressed voltage, may then be treated as the resultant of a major component which leads the voltage by $\pi/2$ radians and a minor component in phase with the voltage. Even for air-dielectric capacitors the in-phase component of current

[9] Duralumin is a class of copper-bearing aluminum alloys. A typical analysis is Cu—4.5 per cent, Mn—0.8 per cent, Mg—0.4 per cent, Si—0.8 per cent.

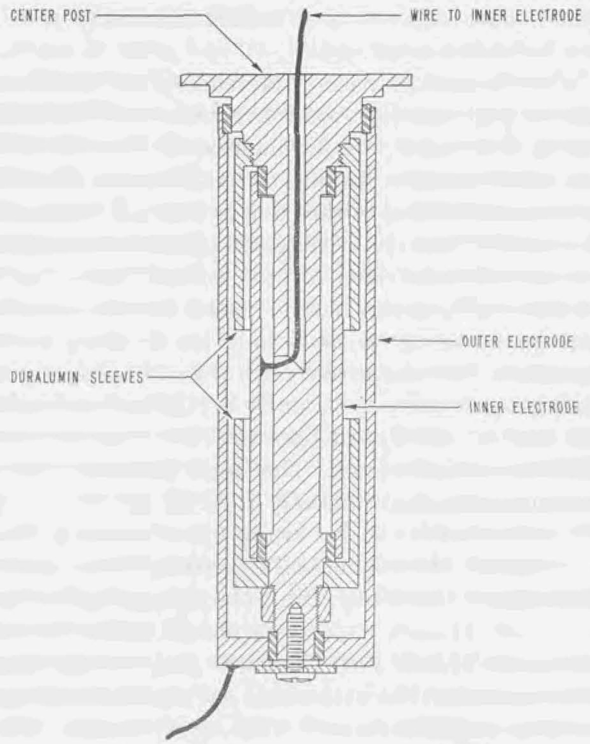

Fig. 7—Section of temperature compensated 1-pf capacitor.

may be only four or five orders of magnitude smaller than the quadrature component, and it is necessary therefore that any device for the precise comparison of capacitors be provided with a means of compensating their loss components of current.

The balance condition for the bridge network is satisfied when the current from one capacitor to the detector junction is equal in magnitude and phase to that from the detector junction to the other capacitor. Hence an unbalance which results from lack of equality between the in-phase components of these currents may be corrected by injecting an in-phase current of proper sign and magnitude at the detector junction. This is accomplished as shown in Fig. 8.

The reactance of the parallel combination of C_1 and C_2 is so small relative to R that the current i_d is very nearly in phase with the voltage e_1. It is brought exactly in phase by means of a small trimming capacitor C_t connected from the midpoint of R to ground. The magnitude of i_d is determined by the voltage applied to R, the value of R, and the relative values of C_1 and C_2, which act as a current divider.[10]

The decade voltage divider is a tapped inductor with four separate windings connected to linear switches of the type described in the preceding section. This is shown in Fig. 9. High accuracy of voltage division is

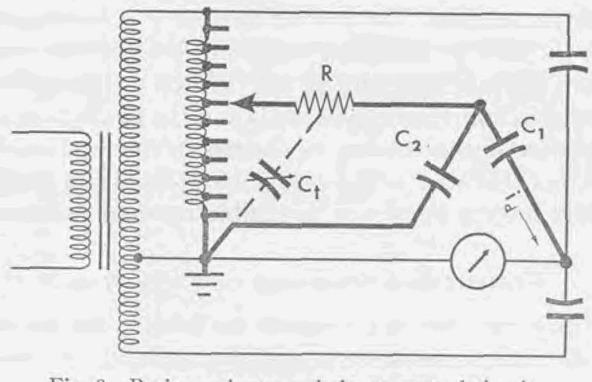

Fig. 8—Basic conductance balance control circuit.

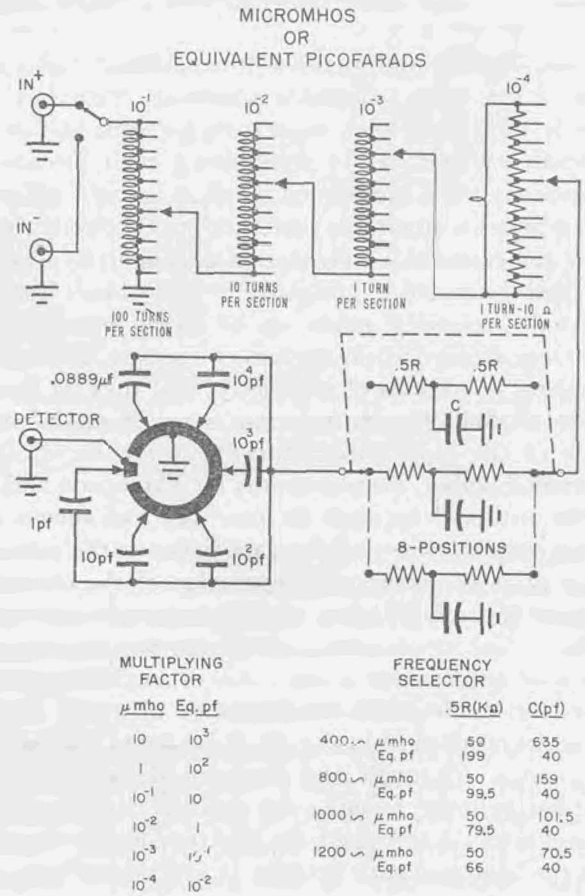

Fig. 9—Details of conductance balance control circuit.

obtained by winding all turns on the same high permeability core. The Supermalloy 1-mil tape-wound core has a section of 0.25 in^2 and an outside diameter of 2.5 inches. AWG no. 26 copper wire was used for all windings.

If the conductance balance control were to provide a measure of the loss characteristics of capacitors expressed in the common unit of conductance, the micromho, small changes in the phase trimming capacitor C_t would suffice to adapt the unit for use with various frequencies. However, it is often convenient to express the loss characteristic of a capacitor in "equivalent capacitance" units. As the conductance and "equivalent capacitance" are related by the equation $C_{eq}=G/\omega$, a change in frequency changes the relative magnitudes of

[10] The value of C_t is so small relative to the sum of C_1 and C_2 that its effect is negligible on the magnitude of the current injected at the detector branch point. It should be noted that the use of a 0.0889-μf capacitor as the final step of the capacitance current divider results in a multiplying factor on the highest range which departs by 10 per cent from its nominal value. In a later construction a 0.1-μf capacitor has been used and the values of R modified so that the nominal ratio of 10 is preserved for all multiplying factors.

their increments. The desired flexibility is incorporated into the unit by providing frequency selection switching which changes the value of the resistor if the reading is desired in equivalent picofarads and the phase trimming capacitor if the reading is desired in micromhos. The unit is calibrated in the bridge by comparison with resistors having known or negligible residual reactances.

Computable Standard of Capacitance

The sensitivity and range of the bridge, the accuracy of its ratio transformers, and the stability of the air capacitors described above require a better low-value standard of capacitance than has been available heretofore at NBS.

Parallel plate and guard-well capacitors[11] have been used as low-value calculable standards. Plans for materially improving such capacitors by using optical interference techniques in determining their mechanical dimensions were abandoned when it became apparent that a better computable capacitor could be constructed using the electrode configuration suggested by Thompson and Lampard.[12] Lampard[13,14] had shown that an electrode assembly made up of equal right circular cylinders, located at the corners of a square, fulfilled the necessary conditions of symmetry, and that for certain types of small departures from symmetry, the mean value of the cross capacitances was not significantly affected. Further, measurements by Thompson and one of the authors[15] on such an assembly had shown that the re-entrant nature of the gaps between the cylinders made possible substantial separation of the electrodes without appreciable effect on the measured capacitance. In fact, these measurements indicated that the principal source of uncertainty in the value of a cylindrical cross capacitor made from an assembly of right circular cylinders resulted from the use of insulated end sections as guards. Although the mechanical length of the guarded cylinder forming the defined capacitor could be accurately determined, its electrical length extended into the insulated gaps between it and the guarding end sections.

Any departure from exact colinearity of the cylindrical surfaces adjacent to the gap would shift the electrical length and hence the value of the capacitor in the direction of the lower surface. To attain the desired accuracy in computing the value of such a capacitor, a method was required that would eliminate this end effect from the calculation.

Cylindrical gauge bars ("reference-grade" end standards) of high quality were available in a variety of lengths. The added requirements of uniformity and equality of diameter among the various bars in the assembly could be met within one or two ten-thousandths of an inch.[16] The cylindrical cross capacitor, designed as a computable standard for the present work, is constructed of such gauge bars. The end sections are 2-inch gauge bars having an axial hole for bringing shielded connections through to the central guarded electrode. A 2-inch gauge bar is permanently bolted to, but insulated from, each of the guard bars to form a portion of the defined central electrode. The inner bars can either be wrung together mechanically to form a 4-inch central electrode, or each can be wrung to an end of a 10-inch bar to form a 14-inch central electrode. If the guard assembly is not disturbed, the difference in values of the 4-inch and 14-inch capacitors should equal the computed value of a 10-inch capacitor without the need for end corrections. The attainable accuracy should, under favorable circumstances,[17] very nearly equal the accuracy to which the mechanical length of the 10-inch gauge bar can be determined.

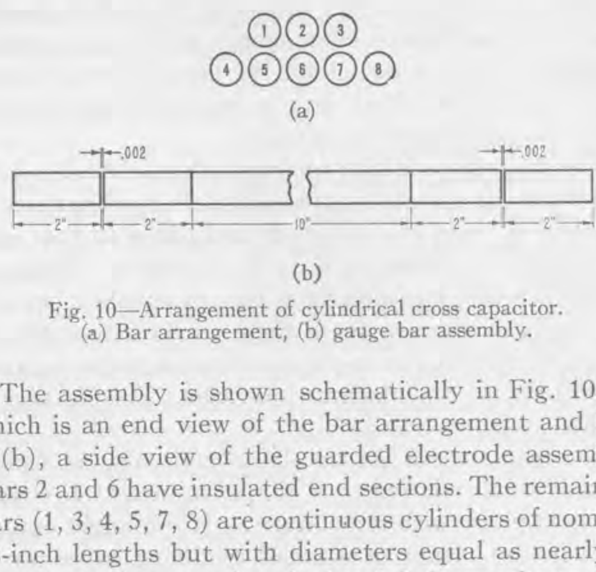

Fig. 10—Arrangement of cylindrical cross capacitor. (a) Bar arrangement, (b) gauge bar assembly.

The assembly is shown schematically in Fig. 10 (a) which is an end view of the bar arrangement and Fig. 10(b), a side view of the guarded electrode assembly. Bars 2 and 6 have insulated end sections. The remaining bars (1, 3, 4, 5, 7, 8) are continuous cylinders of nominal 18-inch lengths but with diameters equal as nearly as possible to the gauge bars in 2 and 6. A capacitance balance for either the 4- or 14-inch assembly can be made with the central section of 2 connected to the detector branch point of the bridge, bars 5 and 7 connected in parallel to one side of the ratio transformer, and the remaining bars, 1, 3, 4, 6, and 8, as well as the guard (end) sections of 2, connected to ground. Capacitances between electrodes 2–5 and 2–7 are in parallel and add directly, doubling the value computed from Lampard's

[11] C. Moon and C. M. Sparks, "Standards for low values of direct capacitance," *J. Res. NBS*, vol. 41, pp. 497–507; November, 1948.

[12] A. M. Thompson and D. G. Lampard, "A new theorem in electrostatics and its application to calculable standards of capacitance," *Nature*, vol. 177, p. 888; May, 1956.

[13] Lampard's theorem may be generalized as follows: If four infinite cylindrical conductors of arbitrary cross sections are assembled with their generators parallel to form a completely enclosed hollow cylinder in such a way that the internal cross capacitances per unit length are equal, then in vacuum these cross capacitances are equal to $\ln_e 2/4\pi^2$ esu/cm.

[14] D. G. Lampard, "A new theorem in electrostatics with applications to calculable standards of capacitance," *Proc. IEE*, vol. 104, pt. C, pp. 271–280; September, 1957.

[15] Informally communicated from NSL, Sydney, Australia.

[16] The assistance of E. J. Schneider of the Engis Equipment Co., Chicago, Ill., and of his principals, the Coventry Gage and Tool Co., Coventry, Eng. in constructing these special gauge bars, is gratefully acknowledged.

[17] It will be apparent that small variations may be introduced by residual misalignment when taking apart and reassembling the end standards forming the defined electrode.

formula for the given length. A second bridge balance can be made using 6 as the detector bar, 1 and 3 in parallel as the line bars, and the remaining bars, 2, 4, 5, 7, 8, together with the end sections of 6 grounded. In this instance the capacitances of 1–6 and 3–6 add in parallel.

The difference between the balance readings taken with 2 and 6 alternatively used as detector bars is a measure of any lack of symmetry in the assembly. The mean of the values is very closely equal to the value computed for the capacitance.[18] A close approximation to symmetry is accomplished by using bars of equal and uniform diameter and by separating them with equal insulating spacers near their ends. The spacers are arranged in the end sections of the assembly so that there is no solid dielectric in the electric field of the calculable central portion. The entire assembly rests on a granite surface plate and is enclosed in a steel housing. This container can be evacuated to a pressure below 0.1 mm Hg, eliminating the need for an air correction to the computed value of capacitance.

A sample set of measurements on the computable capacitor is given in Table I. The individual observations are stated in terms of an arbitrary unit, the mean of the four temperature-compensated capacitors. The value of the cross capacitance is computed in pf, from the measured lengths of the two 10-inch gauge bars. The difference between this computed capacitance and the measured value gives the departure of the arbitrary unit from the absolute unit of capacitance. The uncertainty of this value is believed to be less than 3 ppm.

TABLE I

SAMPLE DATA ON COMPUTABLE CROSS CAPACITOR

MEAN CROSS CAPACITANCE, CORRECTED TO 20°C, REFERRED TO MEAN OF TEMPERATURE-COMPENSATED CAPACITORS

Date	14-Inch Assembly	4-Inch Assembly
7–11–58	1.3897864 Arbitrary Units	
7–11–58	1.3897860	
7–11–58	1.3897855	
7–11–58	1.3897855	
7–14–58	1.3897859	
7–14–58	1.3897866	
7–14–58	1.3897862	
	1.3897860 (Average)	
7–15–58		0.3973392
7–15–58		0.3973395
7–16–58		0.3973400
7–16–58		0.3973400
7–16–58		0.3973391
7–16–58		0.3973399
7–16–58		0.3973402
		0.3973397 (Average)
7–17–58	1.3897864	
7–17–58	1.3897863	
7–21–58	1.3897866	
7–21–58	1.3897862	
7–23–58	1.3897856	
7–23–58	1.3897864	
7–25–58	1.3897875	
	1.3897864 (Average)	

Difference between 14-inch and 4-inch assemblies = 0.9924465.
Computed cross capacitance at 20°C = 0.9924127 pf.
True mean of temperature compensated capacitors
= 1(1–0.0000338) pf.

CONCLUSION

A transformer-ratio bridge for the precise comparison of 3-terminal capacitors has been described. The construction of a cylindrical cross capacitor as a computable standard has also been described. This absolute standard of capacitance is of such a nature as to make full use of the sensitivity and precision available in the bridge. It is expected that the combination of the bridge and the computable standard will make possible assignment of more accurate values to capacitors in the range below 10^4 pf than has been possible in the past at the National Bureau of Standards.

ACKNOWLEDGMENT

It is almost invariably true that the performance of measurement apparatus of high precision depends on the skill and ingenuity of the men who make its mechanical parts. The present bridge is no exception to this rule. The authors wish to acknowledge their indebtedness in this essential matter to the shop personnel working under the direction of D. Kennedy, and especially express their appreciation of the skills and patience of Chidester, Guatney, Graef, Matwey, Pararas, and Stadler.

[18] For example, if the difference in readings amounts to 0.1 per cent, it can be shown that the mean value of the capacitors differs from the computed value by less than 0.1 ppm; more precisely

$$\frac{C_1 + C_2}{2C_0} = 1 + \frac{\ln_e 2}{8}\left(\frac{C_1 - C_2}{C_0}\right)^2,$$

where C_1, C_2 are the measured values and C_0 is the computed value of the capacitor.